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Buoyancy-driven heat transfer of water-based Al_2O_3 nanofluids in a rectangular cavity

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Abstract

In this paper, thermal characteristics of natural convection in a rectangular cavity heated from below with water-based nanofluids containing alumina (Al_2O_3 nanofluids) are theoretically investigated with Jang and Choi's model for predicting the effective thermal conductivity of nanofluids and various models for the effective viscosity. To validate theoretical results, we compare theoretical results with experimental results presented by Putra et al. It is shown that the experimental results are put between a theoretical line derived from Jang and Choi's model and Einstein's model and a theoretical line from Jang and Choi's model and Pak and Cho's correlation. In addition, the effects of the volume fraction, the size of nanoparticles, and the average temperature of nanofluids on natural convective instability and heat transfer characteristics of water-based Al_2O_3 nanofluids in a rectangular cavity heated from below are theoretically presented. Based on the results, this paper shows that water-based Al_2O_3 nanofluids is more stable than base fluid in a rectangular cavity heated from below as the volume fraction of nanoparticles increases, the size of nanoparticles decreases, or the average temperature of nanofluids increases. Finally, we theoretically show that the ratio of heat transfer coefficient of nanofluids to that of base fluid is decreased as the size of nanoparticles increases, or the average temperature of nanofluids is decreased.

1. Introduction

As diverse industrials including microelectronics, transportation, and manufacturing become more advanced, cooling technology is one of the most important challenges [1]. The recent discovery of "nanofluids", which is a new kind of fluid suspension consisting of uniformly dispersed and suspended nanometer-sized (10–50 nm) particles and fibers in base fluid, proposes next approach as cooling technology. This is because nanofluids have fascinating features. One of them is that nanofluids have anomalous high thermal conductivity at very low nanoparticles concentration [2–7] and considerable enhancement of forced convective heat transfer [8–11]. So many investigators have experimentally studied flow and thermal characteristics of nanofluids. Especially, to understand buoyancy-driven heat transfer of nanofluids in a rectangular cavity several investigations have been theoretically and experimentally conducted.

Putra et al. [12] conducted the experiment for observation on the natural convective characteristics of water-based Al_2O_3 nanofluids. They reported that the presence of nanoparticles suspended in base fluid systematically deteriorates the natural convective heat transfer with increasing nanoparticle concentration. The degradation of natural convective heat transfer with increasing particle concentration characterized by decreasing the Nusselt number for a given Rayleigh number was experimentally observed. However, they did not clearly explain why natural convective heat transfer in a cavity is decreased with the increment of the volume fraction of nanoparticles. Kim et al. [13] analytically researched the convective instability driven by buoyancy and heat transfer characteristics of nanofluids with theoretical models which are used to estimate properties of nanofluids. They chose both Einstein's model [14] and Brinkman's model [15] for predicting the

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Nomenclature

C_P	specific heat (kJ/kg K)
d	equivalent diameter (m)
f	volume fraction
g	gravitational acceleration (m/s^2)
h	heat transfer coefficient (W/m^2 K)
H	height of a rectangular cavity (m)
k	thermal conductivity (w/m K)
Nu	Nusselt number
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number
T	temperature (K)

Greek symbols

α	thermal diffusivity (m^2/s)
β	volumetric expansion coefficient (K^{-1})
κ	constant related to Kaptiza resistance
μ	viscosity (N s/m^2)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)

Subscripts

eff	nanofluids
f	base fluid
nano	nanoparticle

effective viscosity of nanofluids, and both Hamilton and Crosser's model [16] and Bruggeman's model [17] for the effective thermal conductivity of nanofluids. However, it is reported that the models used by Kim et al. [13] cannot predict the effective viscosity and the effective thermal conductivity of water-based Al_2O_3 nanofluids [6,18,19]. Khanafer et al. [20] numerically investigated buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. In their numerical model, they utilized Brinkman's model [15] for evaluating the effective viscosity of nanofluids, and Wasp's model [21] for the effective thermal conductivity. In comparison with results from Kim et al. [13], although they used the same model for the effective viscosity and the similar model for the effective thermal conductivity, their numerical results are different from theoretical results of Kim et al. [13]. In addition, they show that as the volume fraction of nanoparticles increases, velocity components of nanofluids in a cavity increase as a result of an increase in the energy transport through the fluid. So, their paper shows the Nusselt number for the natural convection of nanofluids is increased with the volume fraction. These results are not consistent with experimental results presented by Putra et al. [12].

In this paper, as shown in Fig. 1, we investigate natural convective instability and heat transfer characteristics of water-based Al_2O_3 nanofluids in a rectangular cavity heated from below. To obtain the properties of nanofluids we use following models: Jang and Choi's model [6] well

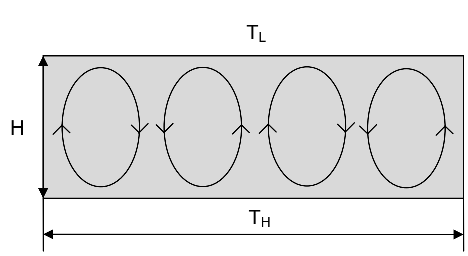

Fig. 1. Natural convection (Benard convection) in a rectangular cavity heated from below.

predicting characteristics of the effective thermal conductivity of nanofluids, Einstein's model [14], Brinkman's model [15], Brownian motion effect's model [22], and an experimental correlation suggested by Pak and Cho [19] for the effective viscosity, and Mixing theory [23] for effective density and specific heat. We compare natural convective heat transfer characteristics resulted from our theoretical model in a rectangular cavity with experimental data of Putra et al. [12] to validate theoretical results. Moreover, we try to explain why natural convective heat transfer in a cavity is decreased with increasing the volume fraction of nanoparticles. Finally we present the effects of the volume fraction, the size of nanoparticles, and the average temperature of nanofluids on natural convective instability and heat transfer characteristics of water-based Al_2O_3 nanofluids in a rectangular cavity heated from below.

2. Instability of natural convection in a rectangular cavity

Natural convection of classical fluid in a rectangular cavity heated from below has been considered by many investigators [24–31]. The Rayleigh number, the most important dimensionless parameter indicating onset of natural convection, is known as the ratio of the buoyancy force to the viscous force acting on the fluids. In the case without Marangoni effect [32] which is related to surface tension at the interface and Soret effect [33] which indicates coupling effect between temperature gradient and mass concentration gradient, the critical value of the Rayleigh number for a normal Newtonian fluid in a rectangular cavity heated from below is given by

$$Ra_{H,C} = \frac{g\beta\Delta TH^3}{\alpha\nu} = \frac{g\beta\Delta TH^3}{k_f\nu}\rho C_P = 1708 \qquad (1)$$

where C_P, g, k_f, β, ν, and ρ are specific heat, gravitational acceleration, thermal conductivity of fluids, volumetric expansion coefficient, kinematic viscosity, and density, respectively. When the Rayleigh number is more than a critical value of $Ra_{H,C} = 1708$ in a rectangular cavity heated from below, buoyancy forces can overcome the resistance imposed by viscous forces and then, there is natural convection within the cavity [34].

Because a Newtonian behavior of water-based Al_2O_3 nanofluids is confirmed by Das et al. [7], we can estimate a critical value of the Rayleigh number in a rectangular cavity heated from below including nanofluids such as water-based Al_2O_3 [13,35].

$$Ra_{\mathrm{H,eff}} = Ra_{\mathrm{H,f}} \frac{k_{\mathrm{f}}}{k_{\mathrm{eff}}} \frac{C_{\mathrm{p,eff}}}{C_{\mathrm{p,f}}} \frac{\mu_{\mathrm{f}}}{\mu_{\mathrm{eff}}} \left(\frac{\rho_{\mathrm{eff}}}{\rho_{\mathrm{f}}}\right)^2 \frac{\beta_{\mathrm{eff}}}{\beta_{\mathrm{f}}} > 1708 \quad (2)$$

where k_{eff}, $C_{\mathrm{p,eff}}$, μ_{eff}, ρ_{eff} and β_{eff} are effective thermal conductivity, specific heat, effective viscosity, density, and volumetric expansion coefficient of nanofluids, respectively. As shown in Eq. (2), in order to evaluate the ratio of the Rayleigh number of nanofluids to that of base fluid, we need to understand relationship of thermal conductivity, specific heat, viscosity, density and volumetric expansion coefficient between nanofluids and base fluid, respectively.

2.1. Effective thermal conductivity of nanofluids

The discovery that thermal conductivity of nanofluids is much higher than predictions was most intriguing. In order to compute the thermal conductivities of fluid/particle mixtures, many researchers have used various theoretical models such as modified traditional models [16,17], nanolayers model [36], Brownian-motion-induced nanoconvection model [6,37,38] and so on. In this paper, Jang and Choi's model [6] which can predict the enhancement of the effective thermal conductivity for nanofluids in terms of nanoparticles' concentration-dependency, nanoparticles' size-dependency and temperature-dependency is used for evaluating the effective thermal conductivity of nanofluids. This is because the effective thermal conductivity models used by Kim et al. [13] and Khanafer et al. [20] cannot predict the effects of the size of nanoparticles and the average temperature of nanofluids on the enhancement of the effective thermal conductivity, although the models are able to predict the effect of the volume fraction of nanoparticles on the enhancement of the effective thermal conductivity of nanofluids. The effective thermal conductivity of nanofluids presented by Jang and Choi [6] is expressed as

$$\frac{k_{\mathrm{eff}}}{k_{\mathrm{f}}} = (1-f) + \kappa_{\mathrm{kapitza}} \frac{k_{\mathrm{nano}}}{k_{\mathrm{f}}} f + C_1 \frac{3d_{\mathrm{f}}}{d_{\mathrm{nano}}} Re^2_{d_{\mathrm{nano}}} Pr_{\mathrm{f}} f \quad (3)$$

where C_1, $\kappa_{\mathrm{kapitza}}$, f, and k_{nano} are a proportional constant, a constant related to Kapitza resistance, volume fraction and thermal conductivity of nanoparticle, respectively.

2.2. Effective viscosity of nanofluids

The ratio of effective viscosity of nanofluids to that of base fluid is calculated by four models.

Einstein's model [14]

$$\frac{\mu_{\mathrm{eff}}}{\mu_{\mathrm{f}}} = (1 + 2.5f) \quad \text{for } f < 0.05 \quad (4)$$

Brinkman's model [15]

$$\frac{\mu_{\mathrm{eff}}}{\mu_{\mathrm{f}}} = \frac{1}{(1-f)^{2.5}} \quad (5)$$

Brownian motion effect's model [22]

$$\frac{\mu_{\mathrm{eff}}}{\mu_{\mathrm{f}}} = 1 + 2.5f + 6.17f^2 \quad (6)$$

Pak and Cho's Correlation [19]

$$\frac{\mu_{\mathrm{eff}}}{\mu_{\mathrm{f}}} = 1 + 39.11f + 533.9f^2 \quad (7)$$

Fig. 2 shows that theoretical results from above models, (4)–(7) are compared with experimental results from previous investigations on the effective viscosity of Al_2O_3 nanofluids. As shown in Fig. 2, while the results of Einstein's model [14], Brinkman's model [15] and Brownian motion effect's model [22] are similar, those of Pak and Cho's correlation [19] present relatively lager value under the fixed volume fraction. The experimental results of Wang et al. [39] are measured by a viscometer, and those of Lee at al. [18] are obtained by measuring the pressure drop and the volume flow rate in a capillary tube. As shown in Fig. 2, the experimental results are less than prediction of Pak and Cho's correlation [19] and larger than others [14,15,22]. So, in this paper to understand buoyancy-driven heat transfer of water-based Al_2O_3 nanofluids in a rectangular cavity we use two models for calculating the effective viscosity of nanofluids. One is Einstein's model [14] and the other is Pak and Cho's correlation [19].

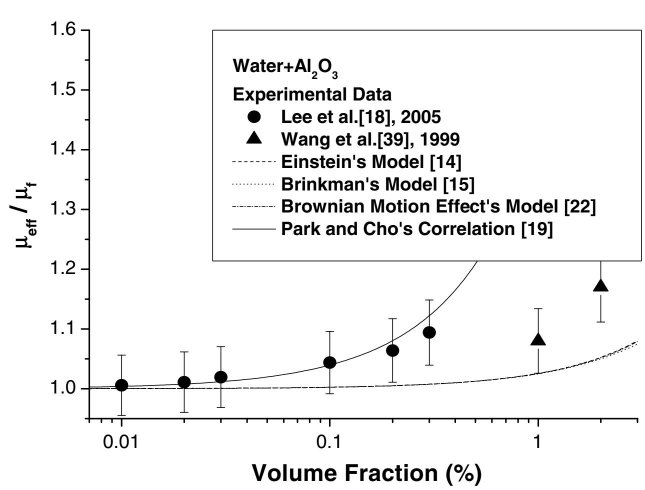

Fig. 2. Comparison between experimental data and theoretical results.

2.3. Specific heat, density and volumetric expansion coefficient of nanofluids

Specific heat, density and volumetric expansion coefficient of nanofluids are predicted by mixing theory [13,23,35].

$$\frac{C_{p,eff}}{C_{p,f}} = (1-f) + f\frac{C_{p,nano}}{C_{p,f}} \tag{8}$$

$$\frac{\rho_{eff}}{\rho_f} = (1-f) + f\frac{\rho_{nano}}{\rho_f} \tag{9}$$

$$\frac{\beta_{eff}}{\beta_f} = 1-f \tag{10}$$

Based on Eqs. (2)–(10), the ratio of the Rayleigh number of nanofluids to that of base fluid is given by

$$\begin{aligned}\frac{Ra_{H,eff}}{Ra_{H,f}} &= \frac{k_f}{k_{eff}}\frac{C_{p,eff}}{C_{p,f}}\frac{\mu_f}{\mu_{eff}}\left(\frac{\rho_{eff}}{\rho_f}\right)^2\frac{\beta_{eff}}{\beta_f} \\ &= \left[(1-f) + k_{kapitza}\frac{k_{nano}}{k_f} + C_1\frac{3d_f}{d_{nano}}Re^2_{d_{nano}}Pr_f f\right]^{-1} \\ &\quad\times\left[(1-f)+f\frac{C_{p,nano}}{C_{p,f}}\right]\left[(1-f)+f\frac{\rho_{nano}}{\rho_f}\right]^2(1-f)\left[\frac{\mu_{eff}}{\mu_f}\right]^{-1}\end{aligned} \tag{11}$$

By substituting Einstein's model [14], or Pak and Cho's correlation [19], into Eq. (11), we can obtain the critical value of the Rayleigh number for water-based Al_2O_3 nanofluids as limiting cases. Based on the results, we investigate the effects of the volume fraction, the size of nanoparticle and the average temperature of nanofluids on natural convective instability of water-based Al_2O_3 nanofluids in a rectangular cavity heated from below.

3. Heat transfer characteristics of natural convection in a rectangular cavity

In classical fluid, Globe and Dropkin [40] proposed the heat transfer coefficient of natural convection in a cavity heated from below.

$$Nu_H = \frac{h_f H}{k_f} = 0.069 Ra_{H,f}^{1/3} Pr_f^{0.074}, \quad 3\times10^5 < Ra_{H,f} < 7\times10^9 \tag{12}$$

where h_f and Pr_f are heat transfer coefficient and Prandtl number, respectively. Based on Eq. (12), the Nusselt number for natural convection in a rectangular cavity with water-based Al_2O_3 nanofluids can be driven by [35]

$$Nu_{H,eff} = \frac{h_{eff} H}{k_{eff}} = 0.069 Ra_{H,eff}^{1/3} Pr_{eff}^{0.074}, \quad 3\times10^5 < Ra_{H,eff} < 7\times10^9 \tag{13}$$

where h_{eff} and Pr_{eff} are heat transfer coefficient and Prandtl number for nanofluids, respectively. The Prandtl number of nanofluids can be derived by the definition of the Prandtl number.

$$\frac{Pr_{eff}}{Pr_f} = \frac{\mu_{eff}}{\mu_f}\frac{C_{p,eff}}{C_{p,f}}\frac{k_f}{k_{eff}} \tag{14}$$

With Eqs. (12)–(14), we can theoretically obtain the ratio of heat transfer coefficient of nanofluids to that of base fluid in a rectangular cavity with natural convection.

$$\begin{aligned}\frac{h_{eff}}{h_f} &= \frac{Nu_{eff}}{Nu_f}\cdot\frac{k_{eff}}{k_f} = \frac{k_{eff}}{k_f}\left(\frac{Ra_{H,eff}}{Ra_{H,f}}\right)^{1/3}\left(\frac{Pr_{eff}}{Pr_f}\right)^{0.074} \\ &= \left(\frac{k_{eff}}{k_f}\right)^{0.593}\left(\frac{C_{p,eff}}{C_{p,f}}\right)^{0.407}\left(\frac{\mu_{eff}}{\mu_f}\right)^{-0.259}\left(\frac{\rho_{eff}}{\rho_f}\right)^{2/3}\left(\frac{\beta_{eff}}{\beta_f}\right)^{1/3}\end{aligned} \tag{15}$$

Based on Eq. (15), we investigate the effects of volume fraction, the size of nanoparticles and the average temperature of nanofluids on the ratio of heat transfer coefficient of nanofluids to that of base fluid in a rectangular cavity with natural convection.

4. Validation

Before investigating the effects of the volume fraction, the size of nanoparticle and the average temperature of nanofluids on natural convective instability and heat transfer coefficient of water-based Al_2O_3 nanofluids in a rectangular cavity heated from below, we compare theoretical results using the effective viscosity model and the effective thermal conductivity used in this paper with experimental results presented by Putra et al. [12]. Fig. 3 shows that experimental and theoretical data have similar tendency. Especially, experimental data presented by Putra et al. [12], are put between a theoretical line derived from Jang and Choi's model [6] and Einstein's model [14] and a theoretical line from Jang and Choi's model [6] and Pak and Cho's correlation [19]. Based on theoretical results, we investigate natural convective instability and heat transfer characteristics in terms of engineering important para-

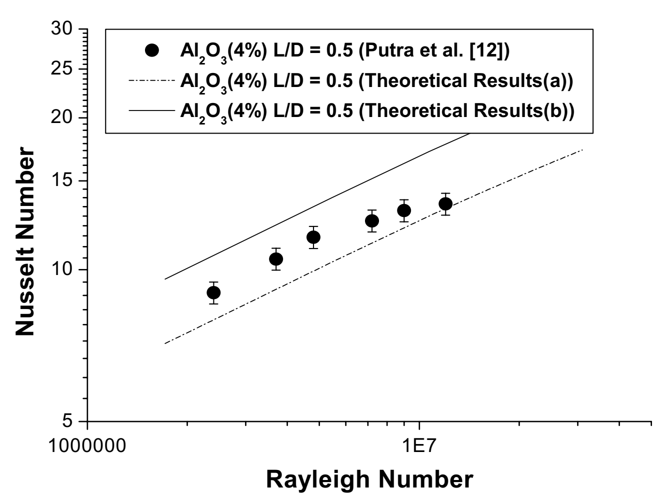

Fig. 3. Comparison between experimental data and theoretical results. (a) Jang and Choi's model [6], Pak and Cho's correlation [19] and (b) Jang and Choi's model [6], Einstein's model [14].

Table 1
Material properties of fluids and nanoparticles [6,34]

	300 K	305 K	310 K	315 K	320 K	325 K
Al_2O_3						
Mean free path (nm)	35	34.43	33.87	33.33	32.81	32.31
Bulk thermal conductivity (W/m K)	42.34	41.66	40.99	40.31	39.62	38.94
Water						
Thermal conductivity (W/m K)	0.613	0.620	0.628	0.634	0.640	0.645
Mean free path (nm)	0.739	0.742	0.745	0.746	0.747	0.747
Viscosity (N s/m^2)	0.855×10^{-3}	0.769×10^{-3}	0.695×10^{-3}	0.631×10^{-3}	0.577×10^{-3}	0.528×10^{-3}
Prandtl number	5.83	5.2	4.62	4.16	3.77	3.42
Density (kg/m^3)	997	995	993	991	989.1	987.3

meters such as the volume fraction, the size of nanoparticles and the temperature using Table 1.

5. Results and discussion

5.1. Natural convective instability in a rectangular cavity

Using the above-mentioned simple correlation, Eq. (11), we investigate natural convective instability of water-based Al_2O_3 nanofluids and the effects of the volume fraction, the size of nanoparticles, and the average temperature of nanofluids on the ratio of the Rayleigh number of nanofluids to that of base fluid. In order to explain the physical meanings of theoretical results, we consider the conservation equations for mass, momentum and energy with scale analysis presented by Bejan [41]. From the scale analysis, we can obtain the friction force per unit mass and the buoyancy force per unit mass under the fixed geometry and temperature difference between top and bottom of a rectangular cavity.

$$\text{Viscous force}: \frac{v}{H^3}\frac{k}{\rho C_p} \ (\text{N/kg}) \tag{16}$$

$$\text{Buoyancy force}: g\beta\Delta T \ (\text{N/kg}) \tag{17}$$

The Rayleigh number indicates the relative magnitude of the buoyancy and viscous forces in the fluid. When buoyancy forces can overcome the resistance imposed by viscous forces and then, natural convection occurs within the rectangular cavity.

As shown in Fig. 4, as the volume fraction of Al_2O_3 nanoparticles increases, the ratio of the Rayleigh number of nanofluids to that of base fluid decreased. The reason is that the viscous force of nanofluids in a rectangular cavity heated from below increases with the volume fraction of nanoparticles due to the enhancement of the effective thermal conductivity of Al_2O_3 nanofluids based on Eq. (16). This phenomenon was experimentally reported by Putra et al. [12]. In particular, under the fixed volume fraction of nanoparticles, the value of Rayleigh number of nanofluids resulted from Pak and Cho's correlation [19] is smaller than that from Einstein's model [14] because the Einstein's model [14] predicts smaller effective viscosity of nanofluids compared with Pak and Cho's correlation [19].

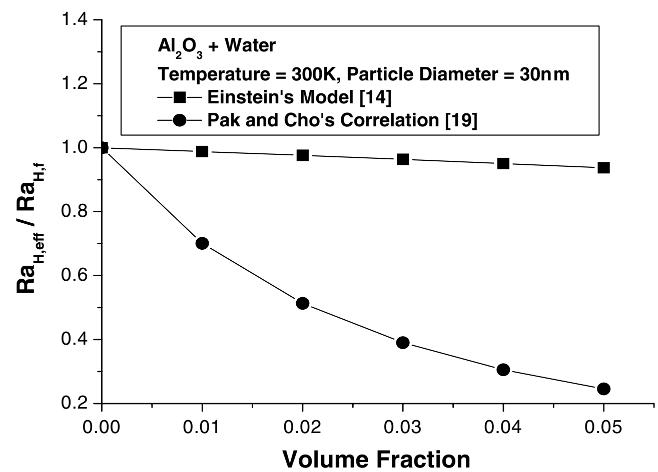

Fig. 4. Effect of the volume fraction on the ratio of the Rayleigh number.

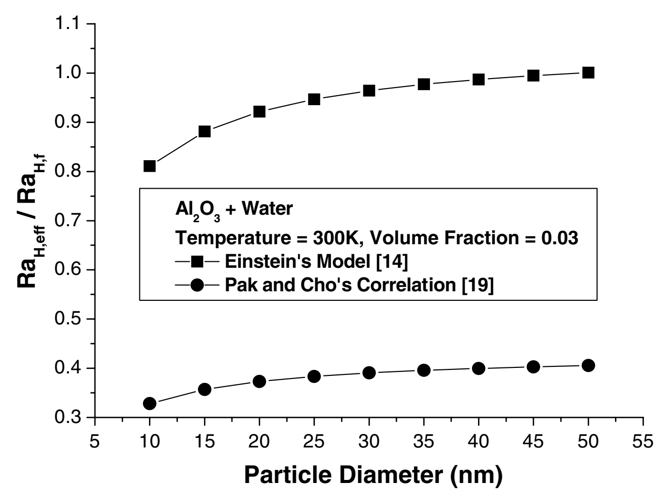

Fig. 5. Effect of the size of nanoparticles on the ratio of the Rayleigh number.

Fig. 5 shows that the ratio of the Rayleigh number of nanofluids to that of a base fluid increases with the size of Al_2O_3 nanoparticles suspended in nanofluids. This is because the effective thermal conductivity of nanofluids is decreased with the increment of the size of nanoparticles [6] and then the viscous force of nanofluids in a rectangular cavity is decreased under the fixed buoyancy force. As shown in Fig. 5 the Rayleigh number of nanofluids is predicted less than that of base fluid in range of the size of nanoparticles from 10 nm to 50 nm under the fixed geometry and temperature difference between top and bottom of a rectangular cavity. The physical meaning for the results is that water-based Al_2O_3 nanofluids with the size of nanoparticles from 10 nm to 50 nm are more stable than base fluid in the cavity under the fixed geometry and temperature difference between top and bottom of a rectangular cavity.

Fig. 6 shows that the ratio of the Rayleigh number of nanofluids to that of base fluid is decreased as the average temperature of nanofluids increases. We can explain the reason as follows: Because the effective thermal conductivity of nanofluids is remarkably increased with temperature [5,6], from Eq. (16) the viscous force of nanofluids in a rectangular cavity heated from below is increased and then the Rayleigh number of nanofluids is decreased. Therefore, the ratio of the Rayleigh number is decreased with the increment of the temperature. This means that nanofluids are more stabilized than base fluid under the given temperature condition.

5.2. Heat transfer characteristics in a rectangular cavity

Based on Eq. (15), the effects of the volume fraction, the size of nanoparticles, and the average temperature of nanofluids on the ratio of heat transfer coefficient of nanofluids to that of base fluid in a rectangular cavity heated from below are theoretically investigated.

Fig. 7 shows the effect of the volume fraction on the ratio of heat transfer coefficient of nanofluids to that of base fluid in a rectangular cavity with natural convection. As shown in Fig. 7, the results from Einstein's model [14] and Pak and Cho's correlation [19] show different trend. This is due to fact that the change of the ratios of thermal conductivity, viscosity, and density is dominant in Eq. (15) according to the volume fraction. In addition, the ratios are increased with the volume fraction. However, the ratio of heat transfer coefficient of nanofluids to that of base fluid is proportional to the ratios of thermal conductivity and density, and inversely proportional to the ratio of viscosity. The ratios of thermal conductivity and density are more dominant compared with the ratio of viscosity calculated from Einstein's model [14]. So the ratio of heat transfer coefficient of nanofluids to that of base fluid is increased with the volume faction as shown in Fig. 7. However, the ratio of viscosity resulted from Pak and Cho's correlation [19] is more dominant than the ratios of thermal conductivity, density. So as the volume fraction increases, the ratio of heat transfer coefficient of nanofluids to that of base fluid is decreased.

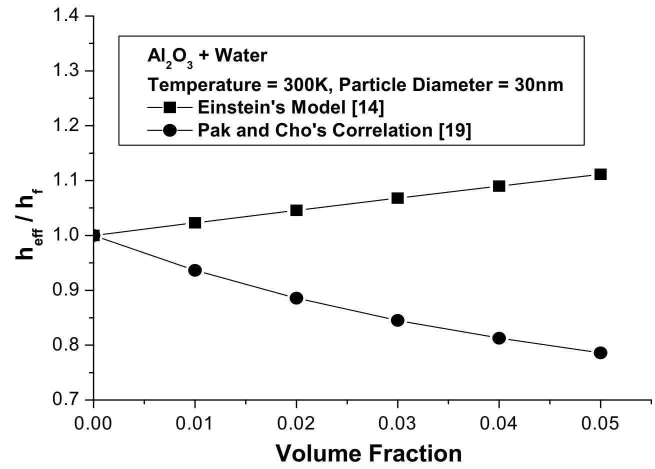

Fig. 7. Effect of the volume fraction on the ratio of heat transfer coefficient.

As shown in Fig. 8, as the size of nanoparticles increases, the ratio of heat transfer coefficient of nanofluids to that of base fluid is decreased. In Eq. (15) the ratio of thermal conductivity is remarkably decreased as the size

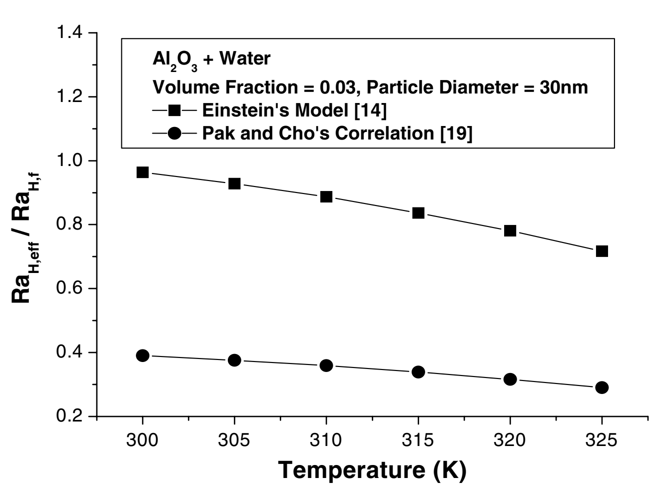

Fig. 6. Effect of the average temperature of nanofluids on the ratio of the Rayleigh number.

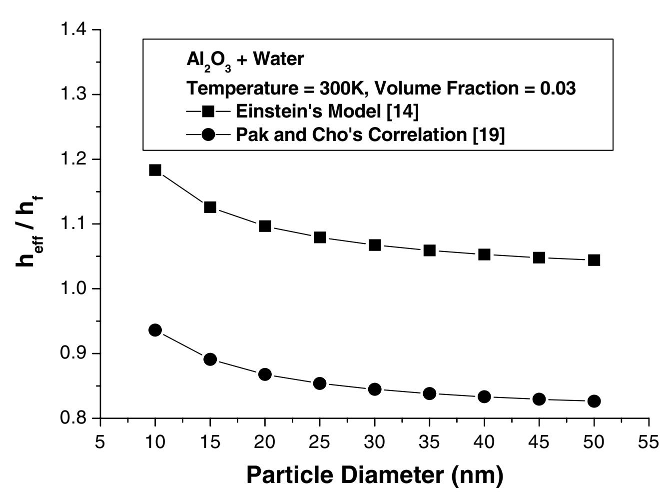

Fig. 8. Effect of the size of particles on the ratio of heat transfer coefficient.

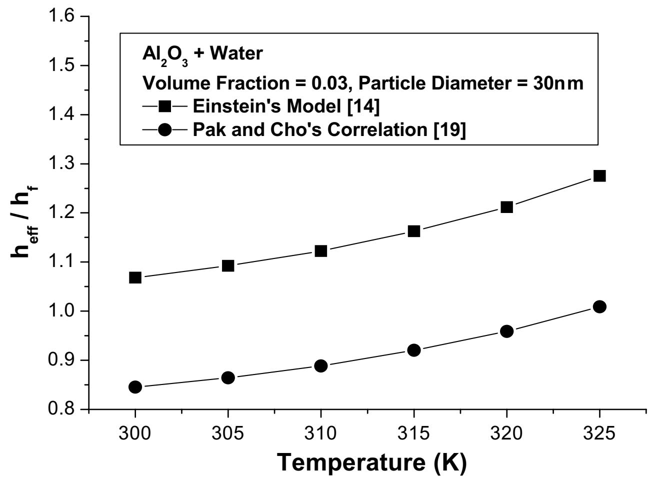

Fig. 9. Effect of the average temperature of nanofluids on the ratio of heat transfer coefficient.

of nanoparticles increases. However, there is no effect of the size of nanoparticles on the ratio of viscosity in Eq. (15). So the ratio of heat transfer coefficient of nanofluids to that of base fluid is decreased as the size of nanoparticles increases.

As shown in Fig. 9, as the average temperature of nanofluids increases, the ratio of heat transfer coefficient of nanofluids to that of base fluid is increased. The reason can be explained as follows: the change of the ratio of thermal conductivity in Eq. (15) becomes dominant as the ratio of thermal conductivity of nanofluids to that of base fluid rapidly increases with the temperature [5,6]. So the ratio of heat transfer coefficient is increased with the average temperature of nanofluids. Especially, in comparison with theoretical results which are estimated by Einstein's model [14] evaluating lower viscosity, the results by Pak and Cho's correlation [19] predict that the heat transfer coefficient of nanofluids is less than base fluid. But Fig. 9 shows that the higher temperature is, the larger the heat transfer coefficient is than base fluid.

6. Conclusion

In this paper, we investigate natural convective instability and heat transfer characteristics of water-based Al_2O_3 nanofluids in the rectangular cavity heated from below using Einstein's model [14] and Pak and Cho's correlation [19] for calculating the effective viscosity of nanofluids and Jang and Choi's model [6] for the effective thermal conductivity. We compare natural convective heat transfer characteristics resulted from our theoretical model in a rectangular cavity with experimental data of Putra et al. [12] to validate theoretical results. We show that experimental results presented by Putra et al. [12] are put between a theoretical line derived from Jang and Choi's model [6] and Einstein's model [14] and a theoretical line from Jang and Choi's model [6] and Pak and Cho's correlation [19]. In addition, the effects of the volume fraction, the size of nanoparticles, and the average temperature of nanofluids on natural convective instability and heat transfer characteristics of water-based Al_2O_3 nanofluids in a rectangular cavity heated from below are theoretically presented. The major findings contained in this paper are as follows: the ratio of the Rayleigh number of nanofluids to that of base fluid is decreased, as volume fraction increases, the size of nanoparticles decreases, or temperature increases. In other words, natural convection of water-based Al_2O_3 nanofluids is more stable than base fluid in a rectangular cavity heated from below, as the volume fraction of nanoparticles increases, the size of nanoparticles decreases, or the average temperature of nanofluids increases. The ratio of heat transfer coefficient of nanofluids to that of base fluid is decreased as the size of nanoparticles is increased, or the average temperature of nanofluids is decreased. In addition, as the viscosity increases the ratio of heat transfer coefficient derived with Einstein's model [14] evaluating lower effective viscosity is increased but the ratio derived with Pak and Cho's correlation [19] is decreased. Based on the results, we can estimate the ratio of heat transfer coefficient exist between the two lines calculated by Einstein's model [14] and Pak and Cho's correlation [19].

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References

[1] J.S. Go, S.J. Kim, G. Lim, H. Yun, J. Lee, I. Song, Y.E. Park, Heat transfer enhancement using flow-induced vibration of a microfin array, Sens. Actuators A 90 (2001) 232–239.

[2] S.U.S. Choi, Enhancing thermal conductivity of fluids with nanoparticles, Develop. Appl. Non Newtonian Flows 1995 (1995) 99–106.

[3] S. Lee, S.U.S. Choi, S. Li, J.A. Eastman, Measuring thermal conductivity of fluids containing oxide nanoparticles, ASME J. Heat Transfer 121 (1999) 280–289.

[4] J.A. Eastman, S.U.S. Choi, W. Yu, L.J. Thompson, Anomalously increased effective thermal conductivity of ethylene glycol-based nanofluids containing copper nanoparticles, Appl. Phys. Lett. 78 (2001) 718–720.

[5] S.K. Das, N. Putra, P. Thiesen, W. Roetzel, Temperature dependence of thermal conductivity enhancement for nanofluids, ASME J. Heat Transfer 125 (2003) 567–574.

[6] S.P. Jang, S.U.S. Choi, The role of Brownian motion in the enhanced thermal conductivity of nanofluids, Appl. Phys. Lett. 84 (2004) 4316–4318.

[7] S.K. Das, N. Putra, W. Roetzel, Pool boiling characteristics of nanofluids, Int. J. Heat Mass Transfer 46 (2003) 851–862.

[8] Y. Yang, Z.G. Zhang, E.A. Grulke, W.B. Anderson, G. Wu, Heat transfer properties of nanoparticle-in-fluid dispersions (nanofluids) in laminar flow, Int. J. Heat Mass Transfer 48 (2005) 1107–1116.

[9] Y. Xuan, Q. Li, Investigation on convective heat transfer and flow features of nanofluids, ASME Trans. J. Heat Transfer 125 (2003) 151–155.

[10] D. Wen, Y. Ding, Experimental investigation into convective heat transfer of nanofluid at the entrance region under laminar flow conditions, Int. J. Heat Mass Transfer 47 (2004) 5181–5188.

[11] S.Z. Heris, S.G. Etemad, M.N. Esfahany, Experimental investigation of oxide nanofluids laminar flow convective heat transfer, Int. Commun. Heat Mass 33 (2006) 529–535.
[12] N. Putra, W. Roetzel, S.K. Das, Natural convection of nano-fluids, Heat Mass Transfer 39 (2003) 775–784.
[13] J. Kim, Y.T. Kang, C.K. Choi, Analysis of convective instability and heat transfer characteristics of nanofluids, Phys. Fluids 16 (2004) 2395–2401.
[14] A. Einstein, Investigation on the Theory of Brownian Motion, Dover, New York, 1956.
[15] H.C. Brinkman, The viscosity of concentrated suspensions and solutions, J. Chem. Phys. 20 (1952) 571–581.
[16] R.L. Hamilton, O.K. Crosser, Thermal conductivity of heterogeneous two-component systems, I&EC Fundam. 1 (1962) 182–191.
[17] D.A.G. Bruggeman, Berechnung Verschiedener Physikalischer Konstanten von Heterogenen Substanzen, I. Dielektrizitatskonstanten und Leitfahigkeiten der Mischkorper aus Isotropen Substanzen, Annal. Phys. Leipzig 24 (1935) 636–679.
[18] J.H. Lee, S.P. Jang, Fluid flow characteristics of Al_2O_3 nanoparticles suspended in water, Trans. SAREK Winter Annual Conf. (2005) 546–551.
[19] B.C. Pak, Y. Cho, Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particle, Exp. Heat Transfer 11 (1998) 151–170.
[20] K. Khanafer, K. Vafai, M. Lightstone, Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids, Int. J. Heat Mass Transfer 46 (2003) 3639–3653.
[21] E.J. Wasp, J P Kenny, R.L. Gandhi, Solid–liquid flow slurry pipeline transportation, Trans Tech Publ., Berlin, 1977.
[22] Davalos-Orozco, Hydrodynamic behavior of suspension of polar particles, Encyclopedia Surface Colloid Sci. 4 (2005) 2375–2396.
[23] J.M. Smith, H.C. Van Ness, Introduction to Chemical Engineering Thermodynamics, McGraw Hill, New York, 1987.
[24] E.R.G. Eckert, W.O. Carlson, Natural convection in an air layer enclosed between two vertical plates with different temperature, Int. J. Heat Mass Transfer 2 (1961) 106–120.
[25] A.E. Gill, The boundary layer regime for convection in a rectangular cavity, J. Fluid Mech. 90 (1979) 561–568.
[26] S. Ostrach, Natural convection in enclosures, in: J.P. Hartnett, T.F. Irvine (Eds.), Advanced in Heat Transfer, 8, Academic Press, New York, 1972, pp. 161–227.
[27] I. Catton, Natural convection in enclosures, in: Proceedings of the 6th International Heat Transfer Conference Toronto Canada, vol. 6, 1978, pp. 13–31.
[28] S.H. Yin, T.Y. Wung, K. Chen, Natural convection in an air layer enclosed within rectangular cavities, Int. J. Heat Mass Transfer 21 (1978) 308–315.
[29] K.H. Chung, J.M. Hyun, Transient natural convection in a cavity with walls of finite thickness, Numer. Heat Transfer; Part A: Appl. 32 (1997) 749–767.
[30] A. Dalal, M.K. Das, Natural convection in a rectangular cavity heated from below and uniformly cooled from the top and both sides, Numer. Heat Transfer; Part A: Appl. 49 (2006) 301–322.
[31] M. Lamsaadi, M. Naimi, M. Hasnaoui, M. Mamou, Natural convection in a vertical rectangular cavity filled with a non-Newtonian power law fluid and subjected to a horizontal temperature gradient, Numer. Heat Transfer; Part A: Appl. 49 (2006) 969–990.
[32] J.K. Bhattacharhee, Marangoni convection in binary liquids, Phys. Rev. E 50 (1994) 1198–1205.
[33] A. Postelnicu, Influence of a magnetic field on heat and mass transfer by natural convection from vertical surface in porous media considering Soret and Dufour effects, Int. J. Heat Mass Transfer 47 (2004) 1467–1472.
[34] F.P. Incropera, D.P. Dewitt, Fundamentals of Heat and Mass Transfer, fifth ed., John Wiley & Sons, 2002.
[35] S.P. Jang, S.U.S. Choi, Free convection in a rectangular cavity (benard convection) with nanofluids, IMECE, 2004-61054.
[36] W. Yu, S.U.S. Choi, The role of interfacial layers in the enhanced thermal of nanofluids: a renovated Hamilton–Crosser model, J. Nanoparticles Res. 6 (2004) 355–361.
[37] J. Koo, C. Kleinstreuer, A new thermal conductivity model for nanofluids, J. Nanoparticles Res. 6 (2004) 577–588.
[38] R.S. Prahser, P. Bhattacharya, P.E. Phelan, Thermal conductivity of nanoscale colloidal solution, Phys. Rev. Lett. 94 (2005) 025901-1–025901-4.
[39] X. Wang, X. Xu, S.U.S. Choi, Thermal conductivity of nanoparticle-fluid mixtures, J. Thermophys. Heat Transfer 13 (1999) 474–480.
[40] S. Globe, D. Dropkin, Natural convection in enclosures, in: Proceedings of the 6th International Heat Transfer Conference, Toronto, Canada, vol. 6, 1971, pp. 13–31.
[41] A. Bejan, Convective Heat Transfer, second ed., John Wiley & Sons, New York, 1995, Chapter 4.